



Province of the
EASTERN CAPE
EDUCATION

**NATIONAL
SENIOR CERTIFICATE/
NASIONALE
SENIOR SERTIFIKAAT**

GRADE/GRAAD 12

JUNE/JUNIE 2016

**MATHEMATICS P2 / WISKUNDE V2
MEMORANDUM**

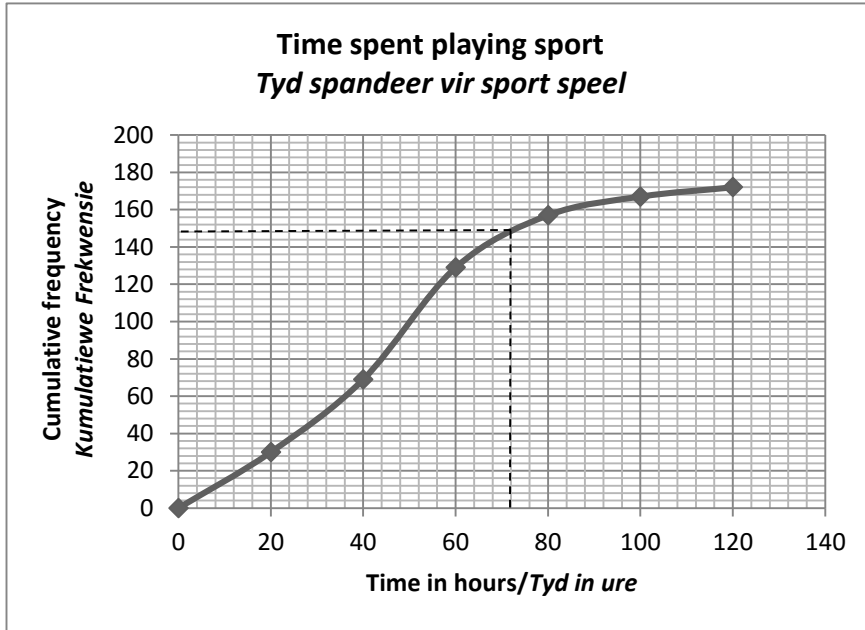
MARKS/PUNTE: 150

This memorandum consists of 8 pages.
Hierdie memorandum bestaan uit 8 bladsye.

QUESTION/VRAAG 1

1.1

Time (hours)/Tyd (ure)	Cumulative Frequency <i>Kumulatiewe Frekwensie</i>
$0 \leq t < 20$	30
$20 \leq t < 40$	69
$40 \leq t < 60$	129
$60 \leq t < 80$	157
$80 \leq t < 100$	167
$100 \leq t < 120$	172



(4)

1.2	$40 \leq t < 60$	✓ answer/antwoord	(1)
1.3	172	✓ answer/antwoord	(1)
1.4	(72;148) $\therefore 172-148 = 24$ learners/ <i>leerders</i>	✓148 ✓24	(2)
1.5	Frequency/ <i>Frekwensie</i> : 30; 39; 60;28;10;5 $\frac{30 \times 10 + 39 \times 30 + 60 \times 50 + 28 \times 70 + 10 \times 90 + 5 \times 110}{172}$ $= \frac{7880}{172}$ $= 45,81$	✓ frequency/ <i>frekwensie</i> ✓ midpoints/ <i>middelpunte</i> ✓ $\frac{7880}{172}$ ✓ answer/antwoord	(4)

[12]

QUESTION/VRAAG 2

2.1	$\bar{x} = \frac{6772}{20}$ $\bar{x} = 338,6$ ml	✓ $\bar{x} = \frac{6772}{20}$ ✓ answer/antwoord	(2)
2.2	2,71 ml	✓✓ answer/antwoord	(2)
2.3	[338,6 – 2,71; 338,6 + 2,71] [335,89; 341.31]	✓✓ interval/interval	(2)

[6]

QUESTION/VRAAG 3

<p>3.1</p>	$m_{AC} = -\frac{1}{3}$ $m_{BD} = 3 \quad \text{[Diagonals of a rhombus]/}$ $\text{[Hoeklyne van ruit/rombus]}$ $y - y_1 = m(x - x_1)$ $y - 9 = 3(x - 3)$ $3x - y = 0$	<p>✓ S ✓ R ✓ subst, m = 3 & (3;9) in eqn. / verv. m = 3 & (3;9) in verg.</p>	<p>(3)</p>
<p>3.1.2</p>	$x + 3y = 10 \dots\dots\dots (1)$ $3x - y = 0 \dots\dots\dots (2)$ $3x + 9y = 30 \dots\dots\dots (3) \quad (1) \times 3$ $10y = 30 \quad (3) - (2)$ $y = 3$ $x + 3(3) = 10$ $x = 1$ $K(1; 3)$	<p>✓ equating two eqns / gelyk stel van twee vergs. ✓ simplification / vereenvoudiging ✓ y = 3 ✓ x = 1</p>	<p>(4)</p>
<p>3.1.3</p>	$\left[\frac{x+3}{2}; \frac{y+9}{2} \right] = [1; 3]$ $\frac{x+3}{2} = 1 \quad \frac{y+9}{2} = 3$ $x+3 = 2 \quad \& \quad y+9 = 6$ $x = -1 \quad \& \quad y = -3$ $B(-1; -3)$	<p>✓ method using midpoint / metode gebruik middelpunt ✓ simplification / vereenvoudiging ✓ coordinates of B / koördinate van B</p>	<p>(3)</p>
<p>3.1.4</p>	$AD = \sqrt{(3-x)^2 + (9-y)^2}$ $\therefore \sqrt{50} = \sqrt{9 - 6x + x^2 + 81 - 18y + y^2}$ $\therefore 50 = x^2 - 6x + y^2 - 18y + 90$ <p>But/Maar: $x = 10 - 3y$</p> $\therefore (10 - 3y)^2 - 6(10 - 3y) + y^2 - 18y + 90 = 50$ $\therefore 100 - 60y + 9y^2 - 60 + 18y + y^2 - 18y + 90 = 50$ $10y^2 - 60y + 80 = 0$ $\therefore y^2 - 6y + 8 = 0$ $\therefore (y - 4)(y - 2) = 0$ $y = 4 \text{ or/of } y = 2$ $x = 10 - 3(4) \text{ or/of } x = 10 - 3(2)$ $x = -2 \quad \text{or/of } x = 4$ $A(-2; 4) \quad C(4; 2)$	<p>✓ subst into eqn dist AD / verv. in verg. afstand AD ✓ subst/verv. $AD = \sqrt{50}$ ✓ subst/verv. $x = 10 - 3y$ ✓ simplification / vereenvoudiging ✓ standard form ✓ values for y ✓ values for x ✓ coordinates</p>	<p>(8)</p>
<p>3.2.1</p>	$m_{PQ} = \frac{8-2}{5-(-3)} = \frac{6}{8}$ $= \frac{3}{4}$	<p>✓ subs into eqn / verv. in verg ✓ answer/antwoord</p>	<p>(2)</p>
<p>3.2.2</p>	$\tan \theta = \frac{3}{4} \quad \theta = 36,9$	<p>✓ tan θ ✓ answer / antwoord</p>	<p>(2)</p>
<p>3.2.3</p>	$y = \frac{3}{4}x + c$ $0 = \frac{3}{4}(8) + c$ $c = -6$ $y = \frac{3}{4}x - 6$	<p>✓ subst/verv. $m = \frac{3}{4}$ ✓ subst/verv. (8;0) ✓ answer/antwoord</p>	<p>(3)</p>

QUESTION/VRAAG 4

4.1	$A(0; y)$ $\therefore p = \frac{0 + 8}{2} = 4$ $\therefore D(4; 4)$	✓ midpt equation / <i>vergelyking</i> ✓ coordinates of D / <i>koördinate van D</i>	(2)
4.2	$A_y = \frac{y+7}{2} = 4$ $A(0; 1)$ $\therefore (x - 0)^2 + (y - 1)^2 = r^2$ $\therefore (4 - 0)^2 + (4 - 1)^2 = r^2$ $\therefore 16 + 9 = r^2$ $\therefore x^2 + (y - 1)^2 = 25$ $\therefore x^2 + y^2 - 2y - 24 = 0$	✓ y-coordinate of A <i>y-koördinaat van A</i> ✓ A(0; 1) ✓✓ subst (0;1) and (4;4) into equation. <i>verv. (0;1) en (4;4) in</i> <i>vergelyking.</i> ✓ r^2	(5)
4.3	$m_{AB} \times m_{FDE} = -1$ [tan radius]/[raaklyn radius] $m_{AB} = \frac{7-4}{8-4} = \frac{3}{4}$ $\therefore m_{FDE} = -\frac{4}{3}$ $(y - 4) = -\frac{4}{3}(x - 4)$ $\therefore y = -\frac{4}{3}x + \frac{28}{3}$	✓ S/R ✓ $m_{AB} = \frac{3}{4}$ ✓ $m_{FDE} = -\frac{4}{3}$ ✓ subst m and (4;4) into eqn / <i>verv. m en (4;4) in</i> <i>vergelyking.</i> ✓ answer/antwoord	(5)
4.4	$x^2 + y^2 = r^2$ $(8)^2 + (7)^2 = r^2$ $\therefore x^2 + y^2 = 113$	✓ subst (8;7) into eqn <i>verv. (8;7) in vergelyking</i> ✓ answer/antwoord	(2)

[14]

QUESTION 5

5.1.1	$= \frac{\sin x \cdot \cos x \cdot \tan x \cdot \cos x}{\sin x \cdot \cos x \cdot (-\tan x)}$ $= -\cos x$	✓ $\sin x$ ✓ $\cos x$ ✓ $\tan x$ ✓ $\cos x$ ✓ $\sin x$ ✓ $\cos x$ ✓ $-\tan x$ ✓ $-\cos x$	(8)
5.2	$\frac{\sin x}{1 + \cos x} + \frac{1 + \cos x}{\sin x} = \frac{2}{\sin x}$ <p>LHS/LK: $\frac{\sin^2 x + (1 + \cos x)(1 + \cos x)}{\sin x (1 + \cos x)}$</p> $\frac{\sin^2 x + 1 + 2 \cos x + \cos^2 x}{\sin x (1 + \cos x)}$ $\frac{2 + 2 \cos x}{\sin x (1 + \cos x)}$ $\frac{2(1 + \cos x)}{\sin x (1 + \cos x)}$ $= \frac{2}{\sin x}$ $= \text{RHS/RK}$	✓ denominator / <i>noemer</i> ✓ numerator / <i>teller</i> ✓ simplification / <i>vereenvoudiging</i> ✓ identity / <i>identiteit</i> ✓ factorisation/ <i>faktorisering</i>	(5)

5.3	$\begin{aligned} \cos 2x &= \cos(x + x) \\ &= \cos x \cdot \cos x - \sin x \cdot \sin x \\ &= \cos^2 x - \sin^2 x \\ &= \cos^2 x - (1 - \cos^2 x) \\ &= 2 \cos^2 x - 1 \end{aligned}$	<ul style="list-style-type: none"> ✓ expansion / uitbreiding ✓ identity / identiteit 	(2)
5.4	$\begin{aligned} \cos 2x + 3 \sin x &= 2 \\ 1 - 2 \sin^2 x + 3 \sin x &= 2 \\ 2 \sin^2 x - 3 \sin x + 1 &= 0 \\ (2 \sin x - 1)(\sin x - 1) &= 0 \\ 2 \sin x = 1 \quad \text{or/of} \quad \sin x - 1 &= 0 \\ \sin x = \frac{1}{2} \quad \text{or/of} \quad \sin x &= 1 \\ x = 30^\circ + n \cdot 360 \quad x = 90^\circ + n \cdot 360 \\ x = 150^\circ + n \cdot 360 ; n \in \mathbf{Z} \end{aligned}$	<ul style="list-style-type: none"> ✓ identity/identiteit ✓ standard form / standaardvorm ✓ factors/faktore ✓ $\sin x = \frac{1}{2}$ $\sin x = 1$ (both eqns/albei vergls) ✓ $x = 30^\circ$ ✓ $x = 150^\circ$ ✓ $x = 90^\circ$ 	(7)
5.5	$\begin{aligned} \sin A \cos B + \cos A \sin B &= \sin(A + B) \\ &= \sin 90^\circ \\ &= 1 \end{aligned}$	<ul style="list-style-type: none"> ✓ identity/identiteit ✓ subst/vervanging ✓ answer/antwoord 	(3)

[25]

QUESTION/VRAAG 6

6.1		<ul style="list-style-type: none"> ✓ shape g vorm g ✓ intercepts afsnitte ✓ min & max value/waarde ✓ shape f vorm f ✓ asymptotes asimptote ✓ intercept afsnit 	(6)
6.2	$-180^\circ < x \leq -90^\circ$ or/of $0^\circ \leq x \leq 90^\circ$	<ul style="list-style-type: none"> ✓ critical values / kritieke waardes ✓ notation / notasie 	(2)

[8]

QUESTION/VRAAG 7

7.1	$\begin{aligned} MN^2 &= PN^2 + PM^2 - 2PN \cdot PM \cos \widehat{MPN} \\ &= 12^2 + 10^2 - 2(12)(10) \cos 126,9^\circ \\ &= 144 + 100 - 240(-0,6) \\ &= 388,1 \\ MN &= 19,7 \text{ m} \\ MN &= AD = 19,7 \text{ m} \end{aligned}$	<ul style="list-style-type: none"> ✓ correct subst into cos rule / korrekte verv. in cos-reel ✓ simplification / vereenvoudiging ✓ answer / antwoord 	(3)
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7.2	$\frac{1}{2}MN \cdot PT = \frac{1}{2}PN \cdot PM \sin \widehat{MPN}$ [Both equal Area of ΔPMN] / [<i>Albei is gelyk aan oppervlakte van ΔPMN</i>] $\frac{1}{2}(10)PT = \frac{1}{2}(12)(10) \sin 126,9^\circ$ $PT = 12 \cdot \sin 126,9^\circ$ $PT = 9,596 \text{ m}$ $= 9,6 \text{ m}$	✓ Equating Area form / <i>Gelykstel van Opp. formules</i> ✓ correct subst / <i>korrekte verv.</i> ✓ simplification / <i>vereenvoudiging</i> ✓ answer / <i>antwoord</i>	(4)
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[7]

QUESTION 8

8.1	Supplementary / <i>Supplementêr</i>	✓ answer / <i>antwoord</i>	(1)
8.2.1	$\widehat{EFO} = 90^\circ$ [tan radius]/[<i>raaklyn radius</i>] $\widehat{EGO} = 90^\circ$ [tan radius]/[<i>raaklyn radius</i>] $\widehat{EFO} + \widehat{EGO} = 180^\circ$ \therefore FOGE is cyclic quad [opp angles supplementary] <i>is 'n koordevierhoek [teenoorst. hoeke supplementêr.]</i>	✓ S ✓ R ✓ S ✓ R ✓ R	(5)
8.2.2	$\widehat{G}_1 = \widehat{H} = x$ [tan chord]/[<i>raaklyn koord</i>] $\widehat{K}_1 = \widehat{H} = x$ [corresp angles/ooreenkmst hoeke; EK FH] $\therefore \widehat{G}_1 = \widehat{K}_1$ EG is a tangent / is 'n raaklyn [angle between line and chord]/ <i>[hoek tussen lyn en koord]</i>	✓ S ✓ R ✓ S ✓ R ✓ R	(5)
8.2.3	$\widehat{O}_1 = 2\widehat{H}$ [angle at centre]/[<i>middelpuntshoek</i>] $= 2x$ $\therefore \widehat{FEG} = 180^\circ - 2x$ [opp angles of cyclic quad]/ <i>[teenoorst. hoeke van koordevierhoek]</i>	✓ S ✓ R ✓ R	(3)

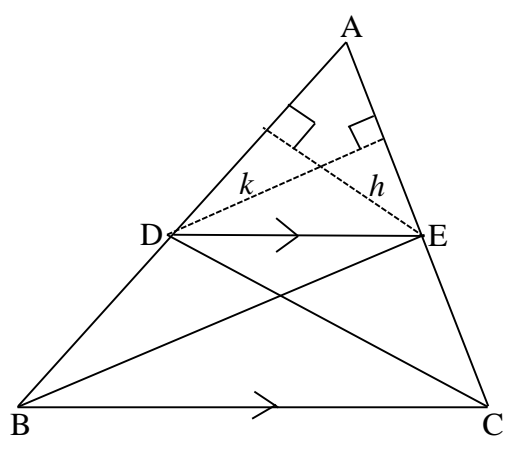
[14]

QUESTION/VRAAG 9

9.1	$\widehat{B}_2 = \widehat{A}_2 = x$ [angles opp equal sides]/[<i>hoeke teenoor gelyke sye</i>] $\widehat{A}_1 = \widehat{B}_2 = x$ [tan chord]/[<i>raaklyn koord</i>] $\widehat{S}_1 = \widehat{A}_1 = x$ [corresp/ooreenkomstig; AD SC] $\widehat{B}_3 = \widehat{A}_2 = x$ [alt int./verwissellend; AD SC] $\widehat{ADC} = \widehat{B}_3 = x$ [ext angle of cyclic quad]/ <i>[buitehoek van koordevierhoek]</i>	✓ S/R ✓ S/R ✓ S/R ✓ S/R ✓ S/R	(5)
9.2	$\widehat{A}_1 = \widehat{ADC}$ [from/vanaf 8.1] AS DC [alt angles equal] / [<i>verw. hoeke gelyk</i>] DA CS [given/gegee] ASCD is a/n parallelogram [opp sides]/[<i>teenoorst. sye </i>]	✓ S ✓ S/R ✓ S ✓ R	(4)
9.3	$\Delta SAB \equiv \Delta ADB$ [A,A,A]/[H,H,H]	✓ ΔSAB	(1)
9.4	$\frac{SA}{AD} = \frac{SB}{AB}$ [from/vanaf 8.3.1] $\therefore AD \cdot SB = SA \cdot AB$ But/Maar AD = SC [ASCD is a/n parallelogram] and/en AB = SA [sides opp equal angles]/[<i>syte teenoor gelyke hoeke</i>] $= DC$ [ASCD is a/n parallelogram] $\therefore SC \cdot SB = DC^2$	✓ S ✓ S ✓ S/R ✓ S/R ✓ R	(5)

[15]

QUESTION/VRAAG 10

10.1.1	In proportion / eweredig	✓ Answer	(1)
10.1.2	 <p>RTP: $\frac{AD}{DB} = \frac{AE}{EC}$</p> <p>PROOF/BEWYS:</p> $\frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\frac{1}{2} \cdot AD \cdot h}{\frac{1}{2} \cdot DB \cdot h} = \frac{AD}{DB}$ $\frac{\text{area } \triangle ADE}{\text{area } \triangle CED} = \frac{\frac{1}{2} \cdot AE \cdot k}{\frac{1}{2} \cdot EC \cdot k} = \frac{AE}{EC}$ <p>But/Maar Area $\triangle BDE = \text{Area } \triangle CED$ (same base and same height)/(dieselfde basis en dieselfde hoogte)</p> $\therefore \frac{\text{area } \triangle ADE}{\text{area } \triangle BDE} = \frac{\text{area } \triangle ADE}{\text{area } \triangle CED}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	<p>✓ ratio of area of / verhouding van opp. van $\triangle ADE : \triangle BDE$</p> <p>✓ $\frac{AD}{DB}$</p> <p>✓ ratio of area of / verhouding van opp. van $\triangle ADE : \triangle CED$</p> <p>✓ $\frac{AE}{EC}$</p> <p>✓ equating two areas / gelykstel van twee oppervlaktes</p>	(5)
10.2.1	$\frac{QT}{TP} = \frac{QW}{WR} \quad \text{prop thm/eweredigh stelling; } TW \parallel VR$ $\therefore \frac{15}{x+2} = \frac{x+4}{x}$ $x^2 + 6x + 8 = 15x$ $x^2 - 9x + 8 = 0$ $\therefore (x-8)(x-1) = 0$ $\therefore x = 8 \text{ or/of } x = 1$	<p>✓ $\frac{QT}{TP} = \frac{QW}{WR}$</p> <p>✓ R</p> <p>✓ substitution / vervanging</p> <p>✓ std form/vorm</p> <p>✓ factors/faktore</p> <p>✓ both values for x / beide waardes van x</p>	(6)
10.2.2	$\frac{PV}{VR} = \frac{PT}{TQ} \quad \text{prop thm/eweredigh stelling ; } TV \parallel QR$ $\frac{PV}{18} = \frac{10}{15}$ $PV = 12 \text{ units/eenhede}$	<p>✓ S/R</p> <p>✓ substitution / vervanging</p> <p>✓ answer/antwoord</p>	(3)

[15]

QUESTION/VRAAG 11

11.1	$D_2 = 90^\circ$ [line from centre to midpoint of chord]/[lyn vanaf die middelpnt van sirkel na middelpnt van koord] In $\triangle ABC$ & $\triangle DOC$ i) $\hat{A} = D_2$ [both equal to 90°]/[beide gelyk aan 90°] ii) $\hat{C} = \hat{C}$ [common/gemeen] $\therefore \triangle ABC \equiv \triangle DOC$ [AAA/HHH]	✓ S/R ✓ S/R ✓ S	(3)
11.2	$\frac{OC}{BC} = \frac{DC}{AC}$ $\triangle ABC \equiv \triangle DOC$ $OC = \frac{DC \cdot BC}{AC}$	✓ S/R	(1)
11.3	$AC^2 = BC^2 - AB^2$ [Pythagoras] $= 30^2 - 18^2$ $= 576$ $AC = 24 \text{ cm}$ $OC = \frac{DC \cdot BC}{AC}$ $= \frac{15 \times 30}{24} = 18,8 \text{ cm}$	✓ S/R ✓ subst in eqn / verv. in verg. ✓ $AC = 24$ ✓ subst / vervang. ✓ answer / antwoord	(5)

[9]

TOTAL/TOTAAL: 150